Hybrid pde models for bio-medical applications

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In this talk, we consider the mathematical modelling and numerical simulation of different bio-medical applications ranging from cerebral aneurysmen to vascular tumor growth and drug response. We combine numerical schemes on different spatial scales and illustrate the mathematical and numerical challenges of the coupling of 0D-1D-3D models as well as of agent-based dynamics.

Firstly, we consider an abstract framework for mixed dimensional approaches. While nonlinear hyperbolic 1D PDE models can effectively be used to describe the flow and transport in the vascular tree, homogenized 3D porous media models can be considered to simulate flow and transport in capillaries and tissue. We introduce a calibrated generic arterial network to generate inflow data for patient specific vasculature and tissue data sets.

Secondly, we comment on the interaction of micro- and macro-scales effects. While agent-based simulation approaches can effectively model cell-cycle effects, the extensions to tissue-scale mechanical interactions is not straightforward. To overcome these limitations, agent based cell dynamics is coupled to a FEM based poro-elastic formulation of the surrounding tissue. This enables us to study the interplay between cellular processes and macroscopic mechanical forces, such as interstitial fluid flow and pressure-induced cell responses.

Thirdly, we consider different aspects and mathematical challenges in case of cerebral aneurysmen. From non-Newtonian 3D blood flow by Lattice Boltzmann methods in patient specific geometries to the endovascular coil embolization by an extended discrete elastic rod model taking into account bending, twisting and self-contact to the formation of clots by a system of reaction-advetion-diffusion equations coupled to the Navier--Stokes system.